

Thermodynamic of universe with a varying dark energy component

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We consider a FRW universe filled by a dark energy candidate together with other possible sources which may include the baryonic and non-baryonic matters. Thereinafter, we consider a situation in which the cosmos sectors do not interact with each other. By applying the unified first law of thermodynamics on the apparent horizon of the FRW universe, we show that the dark energy candidate may modify the apparent horizon entropy and thus the Bekenstein limit. Moreover, we generalize our study to the models in which the cosmos sectors have a mutual interaction. Our final result indicates that the mutual interaction between the cosmos sectors may add an additional term to the apparent horizon entropy leading to modify the Bekenstein limit. Relationships with previous works have been addressed throughout the paper. Finally, we investigate the validity of the second law of thermodynamics and its generalized form in the interacting and non-interacting cosmoses.

I. INTRODUCTION

Since the expanding universe is homogeneous and isotropic on scales larger than about 100-Mpc, it can be modeled by the so-called FRW metric [1]

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right], \quad (1)$$

where $k = 0, \pm 1$ is the curvature constant corresponding to a flat, closed and open universe, respectively. Additionally, $a(t)$ is the scale factor written as $a(t) = a_0 t^{\frac{2}{3(1+\omega)}}$ for $\omega > -1$ and $a(t) = a_0 \exp Ht$ when $\omega = -1$, while $\omega = \frac{p}{\rho}$ is the state parameter of perfect dominated fluid. In addition, $H \equiv \frac{\dot{a}}{a}$ is the Hubble parameter [1]. Moreover, for Phantom regimes ($\omega < -1$) the scale factor is written as $a(t) = a_0(t_{br} - t)^{\frac{2}{3(1+\omega)}}$, where t_{br} is the big rip singularity time, everything will be decomposed to its fundamental constituents at that time, [2]. Additionally, it is shown that one can use the conformal form of this metric to describe the inhomogeneity of the cosmos in scales smaller than 100-Mpc [3]. In the standard cosmology a primary inflationary expansion era is used to get a suitable theoretical description for horizon problem which emerges in the study of Cosmic Microwave Background (CMB) [1]. Observational data signals us a universe with $\dot{a} \geq 0$ and $\ddot{a} \geq 0$ [4–7], which means that we need to modify the gravitational theory [8–10] or considering an unknown source, named dark energy (DE), for describing this phase of expansion [10–12]. A simple model used to explain DE considers an unknown fluid with constant density, pressure and $\omega_D = -1$ called the cosmological constant (CC), and leads to an exponential expansion ($a(t) = a_0 \exp Ht$) [1]. We should note that the current expanding phase of the universe is in full agreement with both of the thermodynamics equilibrium conditions and the rise of complexity content of the universe meaning that the universe may maintain its current expanding phase [13]. More studies on the thermodynamics of DE and the final state of universe can be found in refs. [14–16]. Bearing the primary inflationary era together with the CC model of DE in mind, two difficulties including the *fine tuning* and *coincidence* problems are inevitable [1]. It is also useful to mention here that since the CC model has a satisfactory match to the observational data, it forms a basis for the standard cosmology [1].

Observational data support a DE candidate with varying energy density [17–22]. Indeed, there are various attempts to model the source of the primary and current accelerating eras by introducing a varying model for the DE candidate [9–12, 23–36]. Recently, Lima and co-workers proposed that a universe filled by a dynamical vacuum energy density can avoid the big bang as well as the big crunch singularities, the fine tuning and coincidence problems [36]. Indeed, since the vacuum density is decreased as a function of the Hubble parameter, the Lima's model has enough potential for solving the fine tuning and coincidence problems [36, 37]. Additionally, because in their model, the cosmos began to expand from a primary unstable de-Sitter spacetime, and finally reaches to another eternal de-Sitter spacetime, the horizon problem as well as the big crunch problem are naturally solved [36]. Moreover, It is shown that the ultimate de-Sitter spacetime is in accordance with the thermodynamic equilibrium conditions, and therefore the cosmos may serve its final stage [38]. In this model, the state parameter of the vacuum energy satisfies the $\omega_D = -1$ condition,

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and thus the vacuum energy decays into the other fields confined to the apparent horizon of the FRW universe [36, 39]. It is worthwhile to mention here that the decay of vacuum into the other fields is due to a mutual interaction between the cosmos sectors leading to leave thermal fluctuations into the cosmos in this model [39]. It is a good feature, because observations allows a mutual interaction between the cosmos sectors [40–44]. Thermodynamics of such possible mutual interactions are also studied in various theories of gravity by considering various models for DE [44–48]. In fact, the relation between such possible interactions, coincidence and fine tuning problems and thermal fluctuations attracts more investigators to itself [39, 43, 48, 49].

Similarity between the Black Holes laws and those of thermodynamics motivates us to define a temperature as

$$T = \frac{\kappa}{2\pi}, \quad (2)$$

where κ is the surface gravity of Black Hole [50]. On one hand, for some spacetimes, such as the de-Sitter spacetime, surface gravity and thus the corresponding temperature are negative [50], and therefore we need to define $T = \frac{|\kappa|}{2\pi}$ in order to get the positive values for temperature [51, 52]. Whiles, on the other hand, one can get the Einstein equations on the event horizon of Black Holes (as a causal boundary) by applying the first law of thermodynamics on the event horizon and considering Eq. (2) as a suitable definition for temperature [53–55]. Indeed, it seems that this similarity is much more than a mere resemblance [53–65].

The apparent horizon of the FRW universe, as the marginally trapped surface, is evaluated by

$$\partial_\alpha \zeta \partial^\alpha \zeta = 0 \rightarrow r_H, \quad (3)$$

where $\zeta = a(t)r$, and can be considered as the causal boundary [66–68]. Therefore, One gets [69, 70]

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + \frac{k}{a(t)^2}}}. \quad (4)$$

Moreover, the surface gravity associated with the apparent horizon of the FRW universe can be evaluated by using

$$\kappa = \frac{1}{2\sqrt{-h}} \partial_a (\sqrt{-h} h^{ab} \partial_b \zeta). \quad (5)$$

where $h_{ab} = \text{diag}(-1, a(t)^2)$ [69, 70]. Since the WMAP data indicates a flat universe, from now we set $k = 0$ [1, 2]. Thus, simple calculations lead to

$$\kappa = -H(1 + \frac{\dot{H}}{2H^2}), \quad (6)$$

and therefore

$$T = \frac{\kappa}{2\pi} = -\frac{H}{2\pi}(1 + \frac{\dot{H}}{2H^2}). \quad (7)$$

where we have used Eq. (2) to obtain this equation [51, 68, 71–73]. It is useful to note here that for the FRW universe supported by a fluid with $\rho = -p = \text{constant}$ ($\omega = -1$), $\dot{H} = 0$ and therefore this equation covers the result of de-Sitter spacetime ($T = -\frac{H}{2\pi}$) [50, 51].

In cosmological setups, some authors use various definition of temperature and get the corresponding Einstein equations (Friedmann equations) on the apparent horizon [51, 52, 68, 71–74]. In order to avoid the negative temperature, authors in [52], have defined $T = \frac{H}{2\pi} \simeq \frac{|\kappa|}{2\pi}$ and used the first law of thermodynamics (in the $TdS_A = -dQ$ form) to get the Friedmann equations. In their approach $S_A = \pi \tilde{r}_A^2$ (the Bekenstein limit) and Q are the horizon entropy and energy crossed the apparent horizon, respectively [51]. Indeed, authors argued that the extra minus sign in the first law of thermodynamics is the result of universe expansion leading to decrease the energy of confined fluid together with increase the size of the universe and thus S_A . Therefore, by using original definition of temperature (2) and thus (7), called the Hayward-Kodama temperature [51, 68, 72, 73], together with the $TdS_A = dQ$ form of the first law of thermodynamics we can cover the Friedmann equation. Moreover, it seems that $W = \frac{1}{2}h_{ab}T^{ab}$, where T^{ab} is the energy momentum tensor of fluid which spreads over the cosmos, plays the role of pressure in the dynamics spacetimes and thus the FRW universe [51, 68, 71–75]. Following this argument, authors in [51, 68, 71–74] have used (2) and the work density definition (W) to get the Friedmann equations by applying the first law of thermodynamics ($TdS_A = dQ = dE - WdV$) on the apparent horizon of the FRW universe in various theories of gravity, whiles E is the energy confined to the apparent horizon. It is also shown that Loop Quantum Gravity corrects the

horizon entropy which leads to modify the Friedmann equations on the apparent horizon if one considers (2) together with $TdS_A = dE - WdV$ [63, 75]. The entropy of a self-gravitating system depends on the gravitational theory used to describe the gravity field. Accordingly, it seems that the self-gravitating systems satisfy the Bekenstein limit of entropy in the Einstein general relativity framework. But, since the origin of DE is unknown, it may have either a geometrical or physical origin, one can expect that the DE candidate may affect the horizon entropy. By the same token, it is shown that the ghost dark energy and its generalization, as the dynamics candidates for DE, may also add an additional term to the entropy of various horizons leading to modify the Bekenstein limit [76, 77]. Therefore, it seems that the dynamics model of DE may lead to modify the horizon entropy and thus, the Bekenstein limit. Recently, it is also shown that a mutual interaction between the cosmos sectors may change the horizon entropy [78]. The second law of thermodynamics states that the horizon entropy may meet the $\frac{dS_A}{dt} \geq 0$ condition [79]. Nowadays, thanks to the Bekenstein works [80, 81], it is believed that the rate of the total entropy of a gravitational system should be positive meaning that $\frac{dS_A}{dt} + \frac{dS_{in}}{dt} \geq 0$, while S_{in} is the entropy of confined fluid. The latter is called the general second law of thermodynamics [51, 80, 81]. Comprehensive reviews on the various temperature definitions in cosmological setups, their motivations together with the validity of the first, second and generalized second laws of thermodynamics can be found in refs. [51, 72, 73]. Now, one can ask how a DE candidate and its probable interaction with other parts of cosmos affect the horizon entropy, the second and generalized second laws of thermodynamics?

In this paper, we point to the unified first law of thermodynamics and assume that it is available on the apparent horizon of the flat FRW universe, while T (the horizon temperature) corresponds to the Hayward-Kodama definition of temperature (7) on the apparent horizon of the FRW universe [68, 72, 73], and show that a DE candidate may lead to a new bound for the horizon entropy, while the cosmos sectors do not interact with each other. Additionally, we show that any mutual interaction between the cosmos sectors may also modify the horizon entropy. The relationships with similar works are also studied. Moreover, the results of considering the Cai-Kim temperature are also derived. Finally, the validity of the second law of thermodynamics and its generalization is also addressed. Since the physics behind the Lima's model [36] is completely different from the ordinary models, introducing for describing DE, we point to results of considering this model.

the paper is organized as follows. In the next section, after a brief review on the previous related works, we apply the unified first law of thermodynamics on the apparent horizon of the flat FRW universe, and show that how a dynamic candidate for DE may change the horizon entropy, while the cosmos sectors do not interact with each other. Thereinafter, we generalize our study to the interacting case and get a modification for the horizon entropy due to the mutual interaction between the cosmos sectors. In section (III), we study the validity of the second law of thermodynamics and its generalization. Section (IV) is devoted to a summary and concluding remarks. Throughout this paper, we set $G = c = \hbar = 1$ for simplicity.

II. HORIZON ENTROPY AND THE UNIFIED FIRST LAW OF THERMODYNAMICS

The unified first law of thermodynamics, which is available in some theories of gravity, is written as

$$dE = A\Psi + WdV, \quad (8)$$

where $W = \frac{1}{2}h_{ab}T^{ab}$ and $E = \frac{\zeta}{2}(1 - h^{ab}\partial_a\zeta\partial_b\zeta)|_{\zeta=\bar{r}_A}$ are the work density and the Misner-Sharp energy confined to the apparent horizon, respectively [52, 63–68, 76–78]. In addition, A and Ψ are the area of horizon and the energy supply vector, respectively, and

$$A\Psi = A\psi_a dx^a, \quad (9)$$

while

$$\psi_a = T_a^b \partial_b \zeta + W \partial_a \zeta, \quad (10)$$

is the projection of the total four-dimensional energy-momentum tensor $T_{\mu\nu}$ in the normal direction of the two-dimensional sphere. Consider a perfect fluid source ($T_\nu^\mu = \text{diag}(-\rho_T, p_T, p_T, p_T)$) together with Friedmann equations, by simple calculations we get $E = \rho_T V$,

$$dE - WdV = Vd\rho_T + \frac{p_T + \rho_T}{2}dV, \quad (11)$$

and

$$A\Psi = -AH\zeta\left(\frac{\rho_T + p_T}{2}\right)dt + Aa\left(\frac{\rho_T + p_T}{2}\right)dr, \quad (12)$$

where a is the scale factor. Using the energy-momentum conservation law ($\nabla^\mu T_{\mu\nu} = 0$)

$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0, \quad (13)$$

and $adr = d\zeta - rda$ in rewriting Eq. (12) to obtain

$$A\Psi = Vd\rho_T + \frac{p_T + \rho_T}{2}dV, \quad (14)$$

where $dV = Ad\zeta$ and $A = 4\pi\zeta^2$. By comparing this equation with (11), we get

$$A\Psi = dE - WdV, \quad (15)$$

which is the unified first law of thermodynamics. This result is independent of the number and nature of fluids which support the background spacetime. In addition, one may decompose $T_{\mu\nu}$ into

$$T_{\mu\nu} = T_{\mu\nu}^{DE} + T_{\mu\nu}^m, \quad (16)$$

where $T_{\mu\nu}^{DE}$ and $T_{\mu\nu}^m$ are the energy momentum tensors of DE and the material parts of cosmos (radiation, matter and etc.), respectively. In this situation, it is apparent that $\Psi = \Psi^{DE} + \Psi^m$, $W = W^{DE} + W^m$ and $E = E^{DE} + E^m$, where $E^{DE} = \rho^{DE}V$ and $E^m = \rho^mV$. Therefore, by following the above argument, whenever $\nabla^\mu T_{\mu\nu}^m = \nabla^\mu T_{\mu\nu}^{DE} = 0$, we get

$$A\Psi^m = A\Psi - A\Psi^{DE} = dE^m - W^mdV. \quad (17)$$

δQ (the heat flow crossing the horizon) is determined by the pure matter energy-momentum tensor ($T_{\mu\nu}^m$) as [52, 63–65, 76–78]

$$\delta Q \equiv A\Psi^m. \quad (18)$$

For some gravitational theories, one can use the Clausius relation together with Eq. (7) to get the horizon entropy (S_A) by using [52, 63–65, 76–78]

$$TdS_A = \delta Q \equiv A\Psi^m, \quad (19)$$

which leads to

$$TdS_A = A\Psi^m = A\Psi - A\Psi^{DE} = dE^m - W^mdV, \quad (20)$$

where we have used (17) to get the last equation. It is useful to note that for some theories such as the $f(R)$ gravity, Eq. (19) and thus (20) is not always available [64].

Recently, some authors considered the Hayward-Kodama definition of temperature (7), a universe filled by either a ghost dark energy or its generalized form together with a pressureless matter and use the $TdS_A = A\Psi - A\Psi^{DE}$ relation to get an expression for the entropy (S_A) [76]. Their results show that the entropy of the matter fields differs from the Bekenstein entropy due to the DE effects. They argued that their results are in agreement with the entropy of the apparent horizon in the DGP braneworld model which signals that this approach may be used to get the entropy of apparent horizon in other theories of gravity. This adaptation between these entropies signals that one may find a geometrical interpretation for the origin of the ghost dark energy model (as a DE candidate) by using the DGP braneworld model of gravity. Motivated by this work, Sheykhi extended their work to the apparent horizon of the FRW universe and used the $TdS_A = dE^m - W^mdV$ relation to get the same result as that of ref. [76] for the entropy. Finally, he concludes that the obtained relation for the entropy (S_A) may be interpreted as the corrected relation for the apparent horizon entropy [77]. It is useful to stress here that Eq. (17) clarifies the reason of getting the same result for the horizon entropy by authors in Refs. [76, 77]. Moreover, their results are available only when $\nabla^\mu T_{\mu\nu}^m = \nabla^\mu T_{\mu\nu}^{DE} = 0$ which means that the cosmos sectors do not interact with each other. Recently, by considering the FRW universe in which the cosmos sectors interact with each other, Mitra et al. use the $TdS_A = A\Psi - A\Psi^{DE}$ relation to get the trapping horizon entropy in the Einstein relativity framework. They argued that the obtained relation for the entropy differs from the Bekenstein entropy due to the mutual interaction between the cosmos sectors [78]. In continue, Mitra et al. extended their hypothesis to other different gravity theories [78]. Moreover, it is also shown that a gravitationally induced particle production process as the DE candidate may change the horizon entropy [39]. Bearing the Lovelock theory in mind, some authors used the $TdS_A = A\Psi - A\Psi^{DE}$ relation to get the entropy of the apparent horizon in cosmological setup [65]. Another study including the loop quantum cosmology can be found

in ref [63]. Here, by considering a varying DE candidate, we are going to find a general relation for the entropy of the apparent horizon in both of the interacting and non-interacting cosmoses and investigate the second and generalized second laws of thermodynamics in the Einstein relativity frame work where Eq. (19) and thus (20) are valid.

For this propose, consider the flat FRW universe with Friedmann equation

$$H^2 = \frac{8\pi}{3}(\rho + \rho_D), \quad (21)$$

where ρ_D is the density of dark energy component. In addition, ρ is the density of rest fluids in the cosmos which may include baryonic matters, dark matter and etc, leading to

$$\rho = \rho_{bm} + \rho_{DM} + \dots \quad (22)$$

Therefore, ρ is nothing but ρ^m which is previously introduced. For the sake of simplicity, we omit the m label throughout the paper. From Eq. (21) and the Bianchi identity we get

$$2HdH - \frac{8\pi}{3}d\rho_D = \frac{8\pi}{3}d\rho, \quad (23)$$

and

$$\dot{\rho} + 3H(\rho + p) + \dot{\rho}_D + 3H(\rho_D + p_D) = 0, \quad (24)$$

which is nothing but the energy-momentum conservation law, respectively. In this equation, p_D and p denote the vacuum pressure and the pressure corresponding to the density ρ , respectively. Dot is also denoted as the time derivative. Consider a dark energy candidate with density profile

$$\rho_D = \frac{3\alpha + 3\beta H^2 + 3\gamma H^{2n}}{8\pi}, \quad (25)$$

which converges to CC whiles $\beta = \gamma = 0$. Whiles $n = \frac{1}{2}$, it covers the ghost dark energy model and its generalization for $\alpha = \beta = 0$ and $\alpha = 0$, respectively [23–25]. The $\gamma = 0$ case has been extensively studied in the literatures [26–31]. The results of considering either an arbitrary value for n or optional function of H for $\rho_D = f(H)$ can be found in [32].

Moreover, the cosmological applications of considering model with $\alpha = 0$, $n = \frac{3}{2}$ and $\omega_D = -1$ has also been studied [33, 34]. More similar density profiles for the DE candidate with $\omega_D = -1$ can also be found in [35]. Another attractive case proposed by Lima et al. is obtainable by imposing the $n > 1$ condition together with $\omega_D = -1$ to the density profile of the DE candidate, whenever n is also an integer number [36]. It is useful to note that $E_D = \rho_D V$ and $E = \rho V$ are the energy of dark energy component and the energy corresponding to the density ρ , respectively. Therefore, E is nothing but E^m mentioned previously and we omit the m label for the sake for simplicity.

A. Non-Interacting Models

At the first step we consider a universe in which the cosmos sectors do not interact with each other. Therefore, the energy-momentum conservation law implies (24)

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (26)$$

and

$$\dot{\rho}_D + 3H(\rho_D + p_D) = 0. \quad (27)$$

Substituting (25) into (21) to get

$$\frac{8\pi}{3}\rho = H^2 - \alpha - \beta H^2 - \gamma H^{2n}. \quad (28)$$

Bearing Eq. (26) in mind, by using Eq. (23) we reach at

$$(2H(1 - \beta) - 2n\gamma H^{2n-1})dH = -8\pi H(\rho + p)dt. \quad (29)$$

Using the Hayward-Kodama temperature relation ($-T = \frac{H}{2\pi}(1 + \frac{\dot{H}}{2H^2})$) [51] to obtain

$$-T(2H(1 - \beta) - 2n\gamma H^{2n-1})dH = -4H^2(\rho + p)dt - 2(\rho + p)dH. \quad (30)$$

From Eq. (26), since $E = \rho V$ and $dV = -\frac{4\pi}{H^4}dH$, we get $dE = -4\pi\rho H^{-4}dH - 4\pi H^{-2}(\rho + p)dt$ leading to

$$(\rho + p)dt = -\frac{H^2 dE}{4\pi} - \frac{\rho dH}{H^2}. \quad (31)$$

If we combine this equation with (30) we obtain

$$T(-2H(1 - \beta) + 2n\gamma H^{2n-1})dH = \frac{H^4}{\pi}dE + 2(\rho - p)dH. \quad (32)$$

It is easy to show that this equation can be rewritten as

$$T[(-\frac{2\pi}{H^3}(1 - \beta) + 2n\gamma\pi H^{2n-5})dH] = dE - WdV. \quad (33)$$

In this equation $W = \frac{\rho - p}{2}$ is the work density required for applying a hypothetical displacement $d\tilde{r}_A$ to the apparent horizon [77, 78]. By comparing this result with Eq. (20), one gets

$$dS_A = (-\frac{2\pi}{H^3}(1 - \beta) + 2n\gamma\pi H^{2n-5})dH \quad (34)$$

leading to

$$S_A = \frac{A}{4}(1 - \beta) + \frac{n\gamma\pi^{n-1}}{n-2}A^{2-n}, \quad (35)$$

where $A = 4\pi\tilde{r}_A^2 = \frac{4\pi}{H^2}$ is the area of horizon. Therefore, $\frac{n\gamma\pi^{n-1}}{n-2}A^{2-n}$ is a new term besides the area term. In addition, since the entropy is not an absolute quantity, we have set the integral constant to zero. It is also apparent that, for $n = 2$, entropy is not well-defined. In order to eliminate this weakness, let us restart from Eq. (34), by substituting $n = 2$ and taking integration from that, we get

$$S_A = \frac{A}{4}(1 - \beta) - \frac{\gamma\pi \ln \pi}{2} + \frac{\gamma\pi \ln A}{2} + S_0. \quad (36)$$

Finally, since entropy is not an absolute quantity, one can set $S_0 = \frac{\gamma\pi \ln \pi}{2}$, and gets

$$S_A = \frac{A}{4}(1 - \beta) + \frac{\gamma\pi \ln A}{2}. \quad (37)$$

Therefore, models with $n = 2$ induce a logarithmic correction to the horizon entropy. Logarithmic correction terms have been previously proposed by some authors which either consider the thermal equilibrium and quantum fluctuations in loop quantum gravity framework [83–94] or the thermal fluctuations of system about its thermodynamic equilibrium state [95, 96]. Indeed, logarithmic correction due to the thermal fluctuations are valid in all physical systems [97]. Let us study some choices with $n = \frac{1}{2}$. Bearing Eq. (35) in mind, For a constant vacuum energy density ($\rho_D = \alpha$), we face with the Λ CDM theory and we get $S_A = \frac{A}{4}$ which is in agreement with previous studies [52, 71, 74]. Moreover, for $\alpha = 0$, $\beta = 0$ and $n = \frac{1}{2}$, we have

$$\rho_D = \frac{3\gamma}{8\pi}H, \quad (38)$$

which is the profile density of ghost dark energy model [23, 24]. In this limit, from Eq. (35), we get

$$S_A = \frac{A}{4} - \frac{\gamma}{3\sqrt{\pi}}A^{\frac{3}{2}}, \quad (39)$$

which is in agreement with the ghost dark energy modification to the entropy evaluated previously [76, 77]. Here, we have used the Hayward-Kodama definition of temperature (7) together with the apparent horizon of the FRW universe to get this relation whiles, author in [77], has considered $T = \frac{|\kappa|}{2\pi}$ to get (39) on the apparent horizon. Moreover,

authors in [76] used trapping horizon and the temperature definition $T = \frac{|\kappa|}{2\pi}$ to get this relation. Additionally, equation (25), for $\alpha = 0$ and $n = \frac{1}{2}$, reduces to

$$\rho_D = \frac{3\beta}{8\pi}H^2 + \frac{3\gamma}{8\pi}H, \quad (40)$$

which is the profile density of generalized ghost dark energy model [25, 82]. By considering this profile density we get

$$S_A = \frac{A}{4}(1 - \beta) - \frac{\gamma}{3\sqrt{\pi}}A^{\frac{3}{2}}, \quad (41)$$

as the modification of the generalized ghost dark energy model to the horizon entropy [76]. Although this result is previously obtained by authors in ref. [76], but our derivation is completely different from that of they. Here, we worked on the apparent horizon while they have considered the trapping horizon and found the similar results. In a more general case, for arbitrary functional form of ρ_D , by following the above recipe we get

$$dS_A = \left(-\frac{2\pi}{H^3} + \frac{8\pi^2}{3H^4}\rho'_D\right)dH, \quad (42)$$

where prime denotes derivative with respect to H . Taking integral to obtain

$$S_A = \frac{A}{4} + \frac{8\pi^2}{3} \int \frac{1}{H^4} d\rho_D + C, \quad (43)$$

where C is the integral constant. Therefore, a varying DE candidate imposes a correction term to the horizon entropy in accordance with the first law of thermodynamics and thus, the second term of the RHS of Eq. (43). It is also useful to note that the result of considering CC ($S_A = \frac{A}{4}$) is obtainable by substituting $d\rho_D = 0$ in this equation [71, 74].

Now, let us use the Cai-Kim temperature ($T = \frac{H}{2\pi}$) [52] to get the entropy of apparent horizon. In order to achieve this goal, we follow the approach of authors in ref. [52], where $TdS_A = -dQ$ and $dV = 0$. Using this argument and bearing Eqs. (19) and (20) in mind to reach

$$dS_A = -\frac{V}{T}d\rho. \quad (44)$$

Now, by substituting $d\rho$ from Eq. (23) into this equation, one gets

$$dS_A = -\frac{2\pi}{H^3}dH + \frac{8\pi^2}{3H^4}d\rho_D, \quad (45)$$

which leads to

$$S_A = \frac{A}{4} + \frac{8\pi^2}{3} \int \frac{d\rho_D}{H^4} + C, \quad (46)$$

where C is the integration constant. Therefore, once again, we get a relation for the horizon entropy which is in full agreement with the previous result (43), obtained by considering the Hayward-Kodama temperature.

B. Interacting Models

When the cosmos sectors interact with each other, energy-momentum conservation law implies (24)

$$\dot{\rho} + 3H(\rho + p) = -\dot{\rho}_D - 3H(\rho_D + p_D), \quad (47)$$

meaning that

$$d\rho = -3H(\rho + p)dt - d\rho_D - 3H(\rho_D + p_D)dt. \quad (48)$$

Therefore, by considering Eq. (23) and following the recipe which leads to Eq. (43), we get

$$dS_A = -\frac{2\pi}{H^3}dH - \frac{8\pi^2}{H^3}(\rho_D + p_D)dt, \quad (49)$$

which yields

$$S_A = \frac{A}{4} - 8\pi^2 \int \frac{\rho_D + p_D}{H^3} dt + C, \quad (50)$$

where C is again an integral constant. Therefore, the second term of RHS of this equation is nothing but the entropy correction due to the mutual interaction between the cosmos sectors. For interacting models in which the state parameter of the DE candidate meets the $\omega_D = -1$ condition, and therefore $\rho_D + p_D = 0$, this additional term is zero meaning that the horizon entropy in these models satisfies the Bekenstein limit [80]. For instance, in the model proposed by Lima et al. [36], in which vacuum decays into the other parts of cosmos and $\rho_D + p_D = 0$, the horizon entropy of the flat FRW universe meets the Bekenstein limit [80]. It is in agreement with the initial and final de-Sitter spacetimes of this model, since the horizon of de-Sitter spacetime meets the $S_A = \frac{A}{4}$ condition [52, 71, 74]. Now, let us derive Eq. (50) by using the unified first law of thermodynamics. Bearing the definition of Ψ in mind, simple calculations lead to

$$A\Psi^{DE} = -\frac{3V(\rho_D + p_D)H}{2}dt + \frac{A(\rho_D + p_D)}{2}[d\zeta - \zeta H dt], \quad (51)$$

where we have used the $rda = \zeta H dt$ relation to obtain this equation. It is a matter of calculation to show

$$A\Psi^{DE} = -\frac{4\pi(\rho_D + p_D)}{H^2}\left[1 + \frac{\dot{H}}{2H^2}\right]dt, \quad (52)$$

where we have used $dV = -\frac{3V}{H}\dot{H}dt$ to get this equation. Since we work in the Einstein general relativity framework, Eq. (19) is valid, and thus, simple calculations lead to

$$TdS_A = A\Psi - A\Psi^{DE} = -\frac{H}{2\pi}\left[1 + \frac{\dot{H}}{2H^2}\right]\left(-\frac{2\pi}{H^3}dH - \frac{8\pi^2}{H^3}(\rho_D + p_D)dt\right), \quad (53)$$

where we have used the $A\Psi = T(-\frac{2\pi}{H^3}dH)$ relation, while T is the Hayward-Kodama temperature, in obtaining this relation [51, 68, 72, 73, 78]. It is apparent that this equation is nothing but (49) which leads to Eq. (50).

Our result is in agreement with the recent work by Mitra et al. [78]. Whereas, we have started from the Friedmann equations and considered the apparent horizon as the causal bound, Mitra et al. used the trapping horizon and relation $\delta Q^m \equiv A\Psi - A\Psi^{DE}$ to obtain (50). It is apparent that Eq. (53) clarifies that why both of us get the same results, while, our start points differ from each other.

Finally, let us consider the Cai-Kim temperature to estimate the horizon entropy. In this situation, for an infinitesimal time $dV = 0$, and from Eqs. (14) and (51) we get

$$TdS_A = -A\Psi^m = -A\Psi + A\Psi^{DE} = -V(d\rho + d\rho_D) - \frac{4\pi(\rho_D + p_D)}{H^2}dt, \quad (54)$$

where we have followed the approach of authors in ref. [52] in order to define $TdS_A = -\delta Q$. Now, bearing Eq. (21) in mind, since $T = \frac{H}{2\pi}$, simple calculations lead to

$$S_A = \frac{A}{4} - 8\pi^2 \int \frac{\rho_D + p_D}{H^3} dt + C, \quad (55)$$

where C is an integration constant. Therefore, by using the Cai-Kim temperature and taking into account an infinitesimal time, we get the same result for the horizon entropy as the result obtained in Eq. (50).

III. THE SECOND AND GENERALIZED SECOND LAWS OF THERMODYNAMICS

On one hand, since cosmos is enclosed by the apparent horizon, it forms a closed system and therefore, the entropy of its horizon should increase during the universe expansion meaning that [79]

$$\frac{dS_A}{dt} \geq 0. \quad (56)$$

It is called the second law of thermodynamics. Whereas, on the other hand, the total entropy of the closed systems should be increased. Since cosmos includes spacetime and its contents, which includes the fluids supporting the

geometry of background spacetime, its total entropy consists of two parts including the horizon (S_A) and the confined fluids components (S_{in}) [80, 81]. In fact, the generalized second law of thermodynamics states that the rate of the total entropy of cosmos including the horizon and confined fluids entropies cannot be negative or briefly [80, 81]

$$\frac{dS_A}{dt} + \frac{dS_{in}}{dt} \geq 0. \quad (57)$$

Indeed, the total entropy of gravitational systems should meet (57) [80, 81]. But, here we point to the required conditions for satisfying both of the above criterions.

A. Non-Interacting case

For the non-interacting cases and while ρ_D meets (25), by taking a time derivative of the Friedmann equation (21) and using the energy-momentum conservation law (26) to get the Raychaudhuri equation

$$\dot{H} = -4\pi(\rho + p) \frac{1}{1 - \beta - n\gamma H^{2n-2}}. \quad (58)$$

Since during the cosmos life $\dot{H} < 0$ [1], we get $1 - \beta - n\gamma H^{2n-2} > 0$ leading to $H < (\frac{1-\beta}{n\gamma})^{\frac{1}{2n-2}}$ while $\rho + p > 0$, and $1 - \beta - n\gamma H^{2n-2} < 0$ which yields $H > (\frac{1-\beta}{n\gamma})^{\frac{1}{2n-2}}$ for $\rho + p < 0$. Using Eqs. (30) and (34) to obtain

$$T \frac{dS_A}{dt} = -\frac{4\pi(\rho + p)}{H^2} [1 + \frac{\dot{H}}{2H^2}]. \quad (59)$$

It seems that horizons may satisfy the second law of thermodynamics meaning that the $dS_A \geq 0$ condition should be valid [50, 52, 71, 74]. In order to check the validity of the second law of thermodynamics we insert $T = -\frac{H}{2\pi}(1 + \frac{\dot{H}}{2H^2})$ into this equation, and get

$$\frac{dS_A}{dt} = \frac{8\pi^2(\rho + p)}{H^3}, \quad (60)$$

meaning that the second law of thermodynamics is available for the apparent horizon whiles $\rho + p \geq 0$. This conditions leads to $\omega \geq -1$ for the state parameter ω . Moreover, by combining Eqs. (35) and (27) with together, we get

$$\frac{dS_A}{dt} = -\frac{2\pi\dot{H}}{H^3} (1 + 4\pi \frac{\rho_D + p_D}{\dot{H}}), \quad (61)$$

which means that the second law of thermodynamics is satisfied if $1 + 4\pi \frac{\rho_D + p_D}{\dot{H}} \geq 0$. Finally, the second law of thermodynamics ($\frac{dS_A}{dt} \geq 0$) is met by the horizon component when $\rho_D + p_D \geq -\frac{\dot{H}}{4\pi}$ and $\rho + p \geq 0$ are satisfied, simultaneously. It is useful to mention here that one can get

$$\dot{H} = -4\pi(\rho + p + \rho_D + p_D), \quad (62)$$

by equating Eqs. (61) and (60), which is nothing but the Raychaudhuri equation obtainable by taking time derivative from Eq. (21) and using (24). Therefore, when $\rho + p \geq 0$ and $\frac{\dot{H}}{4\pi} \geq -(\rho_D + p_D)$ are available, then $\rho + p + \frac{\dot{H}}{4\pi} \geq -(\rho_D + p_D)$ is obtainable which is in agreement with the Raychaudhuri equation (62). For the fluids confined to the apparent horizon with total density ρ , the Gibbs law implies [98].

$$T_{in} \frac{dS_{in}}{dt} = \frac{dE}{dt} + p \frac{dV}{dt} = V \frac{d\rho}{dt} - (\rho + p) \frac{4\pi\dot{H}}{H^4}, \quad (63)$$

where $T_{in} \geq 0$ is the temperature corresponding to the confined fluids. Now, using (26) and $V = \frac{4\pi}{3H^3}$ to get

$$T_{in} \frac{dS_{in}}{dt} = -\frac{4\pi(\rho + p)}{H^2} [1 + \frac{\dot{H}}{H^2}], \quad (64)$$

telling us that, for $\rho + p \geq 0$, $\frac{dS_{in}}{dt} \geq 0$ is obtainable when $1 + \frac{\dot{H}}{H^2} \leq 0$ which leads to $H \leq \frac{1}{t}$. The latter means that for the perfect fluids with state parameter ω which either meets the $\omega \leq -1$ or $-\frac{1}{3} \leq \omega$ conditions, $\frac{dS_{in}}{dt} \geq 0$.

Additionally, for a perfect fluid with state parameter $-1 \leq \omega \leq -\frac{1}{3}$, the $\rho + p \geq 0$ condition is satisfied but $\frac{dS_{in}}{dt} \leq 0$. Finally, for a perfect fluid with state parameter ω which satisfies the $-\frac{1}{3} \leq \omega$ condition the generalized second law of thermodynamics ($\frac{dS_A}{dt} + \frac{dS_{in}}{dt} \geq 0$) will be satisfied if the $\rho + p \geq 0$ and $\rho_D + p_D \geq -\frac{\dot{H}}{4\pi}$ conditions are valid. It is useful to mention here that $\omega = -1$ leads to $\frac{dS_A}{dt} = 0$ and $\frac{dS_{in}}{dt} = 0$ meaning that the generalized second law of thermodynamics is marginally satisfied. Moreover, for a more general manner in which ω is not a constant, by using the Raychaudhuri equation, we get

$$1 + \frac{\dot{H}}{H^2} = 1 - 4\pi(\rho + p) \frac{1}{H^2(1 - \beta) - n\gamma H^{2n}}. \quad (65)$$

Thus, $1 + \frac{\dot{H}}{H^2} \leq 0$ leads to

$$H^2(1 - \beta) - n\gamma H^{2n} \leq 4\pi(\rho + p), \quad (66)$$

which indicates that $\frac{dS_{in}}{dt} \geq 0$. Therefore, if this condition is valid, then the generalized second law of thermodynamics will be satisfied.

For the $\rho + p < 0$ case, it is obvious that, from Eq. (60), $\frac{dS_A}{dt} < 0$. In addition, when H meets the $1 + \frac{\dot{H}}{H^2} \leq 0$ condition, $\frac{dS_{in}}{dt} \leq 0$ and thus $\frac{dS_A}{dt} + \frac{dS_{in}}{dt} < 0$ meaning that the generalized second law is not satisfied. Briefly, for a perfect fluid with $\omega < -1$, the generalized second law is not satisfied. If the Hubble parameter satisfies the $1 + \frac{\dot{H}}{H^2} > 0$ condition Eq. (64) leads to $\frac{dS_{in}}{dt} \geq 0$ and therefore, it is legally possible to meet the generalized second law of thermodynamics. Using Eq. (58) to get

$$1 + \frac{\dot{H}}{H^2} = 1 - 4\pi(\rho + p) \frac{1}{H^2(1 - \beta) - n\gamma H^{2n}}. \quad (67)$$

Therefore, the $1 + \frac{\dot{H}}{H^2} > 0$ condition leads to

$$4\pi(\rho + p) < H^2(1 - \beta) - n\gamma H^{2n}. \quad (68)$$

Finally, we can say that if this condition is valid, then $\frac{dS_{in}}{dt} \geq 0$ which may lead to satisfy the generalized second law of thermodynamics.

For the horizon entropy of the flat FRW universe supported by a DE candidate with unknown density profile ρ_D , we can use Eqs. (42) and (23) to obtain

$$\frac{dS_A}{dt} = \frac{8\pi^2(\rho + p)}{H^3}, \quad (69)$$

whenever, it is easy to check that Eqs. (64) and (61) are also valid in this manner. Bearing Eq. (62) in mind, $\dot{H} < 0$ leads to $\rho + p > -\rho_D - p_D$. Similarities with the previous case, in which ρ_D meets Eq. (25), are obvious. In fact, in order to achieve a more detailed resolution about the validity of generalized second law of thermodynamics, we need to know the dependence of either ρ_D or ρ to the Hubble parameter. We should note again that the horizon component satisfies the second law of thermodynamics by ($\frac{dS_A}{dt} \geq 0$) if the $\rho_D + p_D \geq -\frac{\dot{H}}{4\pi}$ and $\rho + p \geq 0$ conditions are met, which is in agreement with the Raychaudhuri equation (62). Moreover, since $\frac{dS_{in}}{dt} \geq 0$ is valid when $-\frac{1}{3} \leq \omega$, the generalized second law of thermodynamics $\frac{dS_A}{dt} + \frac{dS_{in}}{dt} \geq 0$ will be available if the $-\frac{1}{3} \leq \omega$ and $\rho_D + p_D \geq -\frac{\dot{H}}{4\pi}$ conditions are met simultaneously. More studies on the availability of the second law of thermodynamics and its generalization needs to know the exact form of ρ .

B. Interacting Case

For this case, by using (49), we get again

$$\frac{dS_A}{dt} = -\frac{2\pi\dot{H}}{H^3} \left(1 + \frac{4\pi\rho_D(1 + \omega_D)}{\dot{H}}\right). \quad (70)$$

On one hand, when $\rho_D(1 + \omega_D) \geq -\frac{\dot{H}}{4\pi}$, since observationally $\dot{H} < 0$ [1], it seems that $\frac{dS_A}{dt} \geq 0$ is valid everywhere. On the other hand, by combining Eqs. (50), (24) and (23), once again we get

$$\frac{dS_A}{dt} = \frac{8\pi^2(\rho + p)}{H^3}, \quad (71)$$

meaning that $\frac{dS_A}{dt} \geq 0$ is valid everywhere, if $\rho_D(1+\omega_D) \geq -\frac{\dot{H}}{4\pi}$ and $\rho+p \geq 0$ are satisfied simultaneously. Therefore, the quality of validity of the second law of thermodynamics is similar with the non-interacting case. In addition, Eq. (63) leads to

$$T_{in} \frac{dS_{in}}{dt} = -\frac{4\pi(\rho+p)}{H^2} \left[1 + \frac{\dot{H}}{H^2}\right] - V\dot{H}[\rho'_D + 3\frac{H}{\dot{H}}(\rho_D + p_D)], \quad (72)$$

where prime denotes derivative with respect to the Hubble parameter, again. Here, we focus on the model proposed by Lima et al. [36]. In this model, $\omega_D = -1$ while the vacuum density meets Eq. (25). Substituting into the above equation to get

$$T_{in} \frac{dS_{in}}{dt} = -\frac{4\pi(\rho+p)}{H^2} \left[1 + \frac{\dot{H}}{H^2}\right] - V\dot{H} \left[\frac{3\beta H + 3n\gamma H^{2n-1}}{4\pi} \right]. \quad (73)$$

Since $\dot{H} < 0$, the second term of RHS of this equation ($-V\dot{H}\rho'_D$) is positive everywhere and therefore, the validity of $\frac{dS_{in}}{dt} > 0$ and thus the generalized second law of thermodynamics depends on the value of the first term of RHS ($-\frac{4\pi(\rho+p)}{H^2} \left[1 + \frac{\dot{H}}{H^2}\right]$). It is useful to mention here that for a perfect fluid either obeying $\omega \leq -1$ or $-\frac{1}{3} \leq \omega$, the Hubble parameter meets the $H \leq \frac{1}{t}$ condition leading to $1 + \frac{\dot{H}}{H^2} \leq 0$ and thus $\frac{dS_{in}}{dt} > 0$. Moreover, from Eqs. (70) and (71) it is apparent that $\frac{dS_A}{dt} > 0$ when $\omega_D = -1$ and $-1 \leq \omega$, respectively. Therefore, for the flat FRW universe embraced a perfect fluid which satisfies $-\frac{1}{3} \leq \omega$, the generalized second law of thermodynamics is convinced. As again, more studies on the availability of the second law of thermodynamics and its generalization needs to know the exact form of ρ .

IV. SUMMARY AND CONCLUDING REMARKS

Throughout this paper, we considered the FRW universe filled by a DE candidate together a fluid, which is the agent of the other possible sources, which may include the baryonic and non-baryonic matters, enclosed by the apparent horizon of the flat FRW universe. In continue, we proposed a profile density for the DE candidate which covers proposals including CC and dynamic models of DE such as ghost dark energy model, its generalization, the Lima's model and etc. Moreover, by taking into account the Hayward-Kodama definition of the temperature definition of apparent horizon as well as the Friedmann equation, we could find the horizon entropy for models in which the DE candidate does not interact with the other parts of the cosmos. Our study shows that the DE candidate may modify the horizon entropy. We have shown that our formula for entropy (35) is compatible with previous results about the ghost dark energy and its generalization [76, 77]. Indeed, similar result with (41) is reported by authors in ref. [76]. But, our derivation is completely different. Here, we have considered the apparent horizon as the causal bound of the system, while authors in [76] used the trapping horizon as the causal bound to get the associated horizon entropy. In addition, we have generalized our formulation to models in which the DE candidate is an arbitrary unknown function, and showed that the DE candidate may modify the horizon entropy (43) independent of the other parts of cosmos. We have also used the Cai-Kim temperature to get the horizon entropy, and found out that the same result for the horizon entropy is obtainable if one considers an infinitesimal time in which $dV = 0$. Thereinafter, we focused on the models in which the DE candidate interacts with the other parts of cosmos. We found that the mutual interaction between the cosmos sectors may also modify the apparent horizon entropy (50). Our study shows that for models in which $\omega_D = -1$, such as the model proposed by Lima et al. [36], the mutual interaction between the cosmos sectors does not disturb the Bekenstein limit of the horizon entropy. It means that there is no modification to the horizon entropy for interacting models with $\omega_D = -1$ and therefore, $S_A = \frac{A}{4}$ is available in these models. The same as the non-interacting case, we tried to get a relation for the horizon entropy in the interacting models by using the Cai-Kim temperature. Our study shows that the same result as that of obtained by considering the Hayward-Kodama temperature is available for the horizon entropy. Additionally, we pointed to the some required conditions for availability of the second law of thermodynamics and its generalization in the interacting and non-interacting models. Our studies show that for the non-interacting case, while $\rho_D + p_D \geq -\frac{\dot{H}}{4\pi}$, the second law of thermodynamics and its generalization are inevitably valid if the state parameter of other parts of the cosmos satisfies the $-\frac{1}{3} \leq \omega$ condition. It is because $\frac{dS_A}{dt} > 0$ and $\frac{dS_{in}}{dt} > 0$ are separately valid in this situation. Finally, our study shows that for the interacting case with $\omega_D = -1$, $\frac{dS_A}{dt} > 0$ and $\frac{dS_{in}}{dt} > 0$ will be met if $-\frac{1}{3} \leq \omega$ and therefore, the generalized second law of thermodynamics will be available in an unavoidable way.

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- [1] M. Roos, *Introduction to Cosmology* (John Wiley and Sons, UK, 2003).
 - [2] V. Mukhanov, *Physical Foundations of Cosmology* (Cambridge Univ. Press, 2005).
 - [3] N. Riazi, H. Moradpour and A. Amiri, Prog. Theor. Phys. **126**, (2011) 6.
 - [4] A. G. Riess et al., Astron. J. **116**, (1998) 1009.
 - [5] S. Perlmutter et al., Astrophys. J. **517**, (1999) 565.
 - [6] S. Perlmutter et al., Astrophys. J. **598**, (2003) 102.
 - [7] P. de Bernardis et al., Nature **404**, (2000) 955.
 - [8] S. Nojiri and S. D. Odintsov, Phys. Rept. **505**, (2011) 59.
 - [9] S. Capozziello and V. Faraoni, *Beyond Einstein Gravity* (Springer, NY, 2011).
 - [10] K. Bamba, S. Capozziello, S. Nojiri and S. D. Odintsov, Astrophys. Space Sci. **342**, (2012) 155.
 - [11] M. Li, X. D. Li, S. Wang and Y. Wang, THE UNIVERSE. **1**, (2013) 4.
 - [12] S. Nojiri and S. D. Odintsov, Phys. Lett. B **639**, (2006) 144.
 - [13] H. Moradpour and N. Riazi, Int. J. Theor. Phys. DOI 10.1007/s10773-015-2659-2.
 - [14] S. Nojiri and S. D. Odintsov, Phys. Rev. D **70**, (2004) 103522.
 - [15] I. H. Brevik, S. Nojiri and S. D. Odintsov, Phys. Rev. D **70**, (2004) 043520.
 - [16] N. Radicella and D. Pavón, Gen. Relativ. Grav. **44**, (2012) 685.
 - [17] U. Alam, V. Sahni and A. A. Starobinsky, JCAP. **0406**, (2004) 008.
 - [18] D. Huterer and A. Cooray, Phys. Rev. D **71**, (2005) 023506.
 - [19] Y. G. Gong, Int. J. Mod. Phys. D **14**, (2005) 599.
 - [20] Y. G. Gong, Class. Quantum Grav. **22**, (2005) 2121.
 - [21] Y. Wang and M. Tegmark, Phys. Rev. D **71**, (2005) 103513.
 - [22] Y. G. Gong and Y. Z. Zhang, Phys. Rev. D **72**, (2005) 043518.
 - [23] N. Ohta, Phys. Lett. B **695**, (2011) 41.
 - [24] G. Veneziano, Nucl. Phys. B **159**, (1979) 213.
 - [25] A. R. Zhitnitsky, Phys. Rev. D **86**, (2012) 045026.
 - [26] M. Maggiore, L. Hollenstein, M. Jaccard, E. Mitsou, Phys. Lett. B **704**, (2011) 102.
 - [27] J. Solà, Jour. Phys. Conf. Seri. **283**, (2011) 012033.
 - [28] I. L. Shapiro and J. Solà, JHEP. **0202**, (2002) 006.
 - [29] J. Grande, J. Solà and H. Stefancic, Phys. Lett. B **12**, (2006) 040.
 - [30] J. Grande, J. Solà and H. Stefancic, JCAP **0608**, (2006) 011.
 - [31] M. Maggiore, Phys. Rev. D **83**, (2011) 063514.
 - [32] V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D **9**, (2000) 373. Its citations and references therein.
 - [33] J. A. S. Lima and J. M. F. Maia, Phys. Rev. D **49**, (1994) 5597.
 - [34] J. A. S. Lima and M. Trodden, Phys. Rev. D **53**, (1996) 8.
 - [35] J. M. Overduin and F. I. Cooperstock, Phys. Rev. D **58**, (1998) 043506;
P. I. Fomin, P. A. Nakaznoy and S. I. Vilchinskyi, arXiv:gr-qc/0509042v1 and references in there.
 - [36] J. A. S. Lima, S. Basilakos and J. Solà, Mont. Not. R. Astron. Soc. **431**, (2013) 923.
 - [37] J. A. S. Lima, S. Basilakos and J. Solà, arXiv:1412.5196.
 - [38] J. P. Mimoso and D. Pavón, Phys. Rev. D **87**, (2013) 047302.
 - [39] H. Moradpour, M. T. Mohammadi Sabet and A. Ghasemi, arXiv:1505.03324, Accepted for publishing in MPLA.
 - [40] X. D. Xu, B. Wang, and E. Abdalla, Phys. Rev. D **85**, (2012) 083513.
 - [41] E. Abdalla, L. R. Abramo, and J. C. C. de Souza, Phys. Rev. D **82**, (2010) 023508.
 - [42] A. A. Costa, X. D. Xu, B. Wang, E. G. M. Ferreira, and E. Abdalla, Phys. Rev. D **89**, (2014) 103531.
 - [43] X. D. Xu, B. Wang, P. Zhang, and F. A. Barandela, JCAP, **12**, (2013) 001.
 - [44] M. Jamil, E. N. Saridakis and M. R. Setare, Phys. Rev. D **81**, (2010) 023007.
 - [45] R. Herrera, arXiv:1401.8283v1.
 - [46] R. Herrera and N. Videla, arXiv:1406.6305v1.
 - [47] B. C. Paul and A. S. Majumdar, Class. Quant. Grav. **32**, (2015) 11.
 - [48] M. Honarvayan, A. Sheykhi and H. Moradpour, IJMPD, **24**, (2015) 1550048.
 - [49] D. Pavon and W. Zimdahl, Phys. Lett. B **628**, (2005) 206.
 - [50] E. Poisson, *A Relativist's Toolkit*, (Cambridge University Press, UK, 2004).
 - [51] D. W. Tian and I. Booth, arXiv:1411.6547v2.
 - [52] R. G. Cai and S. P. Kim, JHEP **02**, (2005) 050.
 - [53] T. Jacobson, Phys. Rev. Lett. **75**, (1995) 1260.

- [54] T. Padmanabhan, Phys. Rept. **406**, (2005) 49.
- [55] C. Eling, R. Guedens and T. Jacobson, Phys. Rev. Lett. **96**, (2006) 121301.
- [56] M. Akbar and R. G. Cai, Phys. Lett. **B 635**, (2006) 7.
- [57] M. Akbar and R. G. Cai, Phys. Lett. **B 648**, (2007) 243.
- [58] T. Padmanabhan, Class. Quant. Grav. **19**, (2002) 5387.
- [59] T. Padmanabhan, Int. J. Mod. Phys. **D 15**, (2006) 1659.
- [60] A. Paranjape, S. Sarkar and T. Padmanabhan, Phys. Rev. **D 74**, (2006) 104015.
- [61] D. Kothawala, S. Sarkar and T. Padmanabhan, Phys. Lett. **B 652**, (2007) 338.
- [62] T. Padmanabhan and A. Paranjape, Phys. Rev. **D 75**, (2007) 064004.
- [63] R. G. Cai, L. M. Cao and Y. P. Hu, JHEP **08**, (2008) 090.
- [64] R. G. Cai, L. M. Cao, Y. P. Hu and N. Ohta, Phys. Rev. D **80**, (2009) 104016.
- [65] R. G. Cai, L. M. Cao, Y. P. Hu and S. P. Kim, Phys. Rev. D **78**, (2008) 124012.
- [66] S. A. Hayward, S. Mukohyana, and M. C. Ashworth, Phys. Lett. **A256**, (1999) 347.
- [67] S. A. Hayward, Class. Quantum Grav. **15**, (1998) 3147.
- [68] D. Bak and S. J. Rey, Class. Quantum Grav. **17**, (2000) 83.
- [69] A. Sheykhi, B. Wang and R. G. Cai, Nucl. Phys. B **779**, (2007) 1.
- [70] A. Sheykhi, B. Wang and R. G. Cai, Phys. Rev. D **76**, (2007) 023515.
- [71] R. G. Cai and L. M. Cao, Phys. Rev. D **75**, (2007) 064008.
- [72] P. Binétruy and A. Helou, arXiv:1406.1658v1.
- [73] A. Helou, arXiv:1502.04235v1.
- [74] M. Akbar and R. G. Cai, Phys. Rev. D **75**, (2007) 084003.
- [75] A. Sheykhi, Eur. Phys. J. C **69**, (2010) 265.
- [76] C. J. Feng, X. Z. Li and X.Y. Shen. Mod. Phys. Lett. A **27**, (2012) 1250182.
- [77] A. Sheykhi, Can. J. Phys. **92**, (2014) 529.
- [78] S. Mitra, S. Saha and S. Chakraborty, AOP, DOI: 10.1016/j.aop.2015.01.025 (2015).
- [79] S. W. Hawking, Phys. Rev. Lett. **26**, (1971) 1344.
- [80] J. D. Bekenstein, Phys. Rev. D **7**, (1973) 2333.
- [81] J. D. Bekenstein, Phys. Rev. D **9**, (1974) 3292.
- [82] R. G. Cai, Z. L. Tuo, Y. B. Wu, Y. Y. Zhao, Phys. Rev. D **86**, (2012) 023511.
- [83] K. A. Meissner, Class. Quant. Grav. **21**, (2004) 5245;
A. Ghosh and P. Mitra, Phys. Rev. D **71**, (2005) 027502;
A. Chatterjee and P. Majumdar, Phys. Rev. Lett. **92**, (2004) 141301.
- [84] J. Zhang, Phys. Lett. B **668**, (2008) 353;
R. Banerjee and B. R. Majhi, Phys. Lett. B **662**, (2008) 62;
J. High Energy Phys. **06**, (2008) 095;
S. Nojiri and S. D. Odintsov, Int. J. Mod. Phys. A **16**, (2001) 3273.
- [85] R. Banerjee, B. R. Majhi, and S. Samanta, Phys. Rev. D **77**, (2008) 124035;
R. Banerjee and S. K. Modak, J. High Energy Phys. **11**, (2009) 073.
- [86] S. K. Modak, Phys. Lett. B **671**, (2009) 167;
B. R. Majhi, Phys. Rev. D **79**, (2009) 044005;
R. Banerjee and S. K. Modak, J. High Energy Phys. **05**, (2009) 063;
B.R. Majhi and S. Samanta, Ann. Phys. (N.Y.) **325**, (2010) 2410.
- [87] S. W. Hawking, Commun. Math. Phys. **55**, (1977) 133.
- [88] R. Banerjee and B. R. Majhi, Phys. Lett. B **674**, (2009) 218.
- [89] R. K. Kaul and P. Majumdar, Phys. Rev. Lett. **84**, (2000) 5255.
- [90] A. Ghosh and P. Mitra, Phys. Rev. D **71**, (2005) 027502.
- [91] S. Hod, Class. Quant. Grav. **21**, (2004) 97.
- [92] A. J. M. Medved, Class. Quant. Grav. **22**, (2005) 133.
- [93] H. M. Sadjadi and M. Jamil, Euro. phys. Lett. **92**, (2010) 69001.
- [94] D. V. Fursaev, Phys. Rev. D **51**, (1995) 5352;
R. B. Mann and S. N. Solodukhin, Nucl. Phys. B **523**, (1998) 293.
- [95] S. Das, P. Majumdar and R. K. Bhaduri, Class. Quant. Grav. **19**, (2002) 2355.
- [96] B. Wang, C. Y. Lin, D. Pavón and E. Abdalla, Phys. Lett. B **662**, (2008) 1.
- [97] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Oxford, Pergamon, 1969).
- [98] G. Izquierdoand and D. Pavón, Phys. Lett. **B633**, (2006) 420.